It Could Happen

Reporting Category Probability and Statistics
Topic Differentiating between dependent and independent events

Materials
- Sample Events handout (attached)
- Coin
- Number cube
- Deck of playing cards

Vocabulary

probability (earlier grades)

independent events, dependent events (6.16)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Open discussion by asking students how probability is used in real-world applications. Some examples include forecasting weather, ranking college teams for bowl games, and predicting student test results. Explain the importance of understanding probability.
2. To assist students in understanding *dependent events* and *independent events*, read aloud the attached Sample Events scenarios, and have them respond to the comprehension questions following each scenario.
3. Provide students with the formal definitions of dependent events and independent events:
   - **dependent events:** two events so related that the outcome of one event is influenced by the outcome of the other.
   - **independent events**: two events so related that the outcome of one event has no effect on the outcome of the other.

   Connect the definitions to the Sample Events scenarios.
4. In order to explain how to calculate probability, it is recommended to demonstrate examples involving flipping a coin and rolling a number cube. Starting with the coin, ask students how many possibilities one flip has; they should agree that each flip has two possibilities—heads or tails. Next, ask what the probability is of getting heads on a flip and what the probability is of getting tails on a flip. Guide students to comprehend that because each flip has 1 chance for a specific result out of 2 possibilities, each flip has a 1 to 2 probability, or 1:2, or $\frac{1}{2}$. (If further explanation about expressing this ratio is needed, refer

to SOL 6.1). Consequently, the probability of landing on heads is $\frac{1}{2}$, and the probability of landing on tails is $\frac{1}{2}$.

5. Next, ask what the probability is that the coin will land on heads on the first flip and heads again on the second flip? Allow students to predict the answer. After getting responses, explain the **probability formula: *P*(*A* and *B*) = *P*(*A*) · *P*(*B*)**, which is read, "the probability of *A* and *B* equals the probability of *A* times the probability of *B.*" Hence, using the formula, the probability that the coin will land on heads on the first flip and heads again on the second flip is calculated $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. Ask students whether these events are dependent or independent.
6. Present an example with more possibilities, using the number cube. Ask students what the probability is of the cube landing on an even number smaller than 6 three times in a row. Help students reason through this question by asking specific questions as such as:
   - How many faces does a number cube have? (6)
   - What are the numbers shown on the faces of the cube. (1, 2, 3, 4, 5, 6)
   - How many even numbers are on the cube? (3)
   - How many odd numbers are on the cube? (3)
   - How many even numbers smaller than 6 are on the number cube? (2)

   Lead students to reason that because there are two even numbers smaller than 6, the probability of three rolls landing on either of those numbers is calculated $\frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$. Ask students whether these events are dependent or independent.
7. Present an example that uses numbered playing cards. Make a stack of five different cards—a 2, a 4, a 5, a 7, and a 10. Ask students to calculate the probability of drawing a 2 and then a 7. Tell students that after each draw, the card drawn will be put back into the stack and the stack shuffled. Lead students to reason that the probability of drawing a 2 is 1:5, or $\frac{1}{5}$, and the probability of drawing a 7 is 1:5, or $\frac{1}{5}$; therefore, the probability of drawing a 2 and then a 7 is $\frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$. Ask students whether these events are dependent or independent.
8. Present another example using the same numbered playing cards. Ask students to calculate the probability of drawing a 10 and then a 4, but this time, tell them that the first card drawn will *not* be put back into the stack. Lead students to reason again that the probability of drawing a 10 is 1:5, or $\frac{1}{5}$, and then the probability of drawing a 4 is 1:4, or $\frac{1}{4}$; therefore, the probability of drawing a 10 and then a 4 is $\frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$. Ask students whether these events are dependent or independent.
9. Extend the lesson by increasing the number of cards, making sure to include doubles and triples of certain numbers. This allows for variation in the probabilities. In addition,

alternate between dependent and independent events to assist students in distinguishing between the two.

Assessment

- **Questions**
    - What keywords can be associated with dependent events? With independent events?
    - How do probabilities vary based on whether events are dependent or independent?
- **Journal/Writing Prompts**
    - Describe an example of how you could use probability in planning a family activity?

Extensions and Connections (for all students)

- Create other scenarios for calculating probability, using the number cube, coin flipping, same sized objects in a grab bag, etc.

Strategies for Differentiation

- Post the scenarios around the room for a walkabout, and have students jot their responses on the posted papers.
- After reading each scenario aloud, have students respond orally to the comprehension questions.

Sample Events

There are eight people in a business meeting. Eight different gourmet pizza slices are provided for their lunch, and each person must choose one slice. The slice choices are as follows:

1. Pepperoni and Cheese
2. Cheese
3. Sausage and Cheese
4. Pepperoni, Sausage, and Cheese
5. Ham, Pineapples, and Cheese
6. Pepperoni, Mushrooms, and Cheese
7. Mushrooms and Cheese
8. Pepperoni, Bell Peppers, Mushrooms, and Cheese

Are their selections dependent or independent? Why?

A baker prepares six frosting flavors—chocolate, vanilla, lemon, strawberry, cream cheese, and caramel—for decorating cupcakes. She puts all six frosting flavors into the same size buckets, seals them up, and sets them on a shelf. Remembering which bucket contains lemon frosting, she selects lemon on the first day, frosts half the cupcakes, and leaves the lemon frosting off the shelf to finish frosting the cupcakes the next day. The next day, she decides she wants to use the strawberry frosting instead of lemon for the remaining cupcakes, but when she goes to the shelf to select strawberry, she realizes she placed all of the remaining buckets on the shelf without labeling them; therefore, she must select blindly.

Are her selections dependent or independent? Why?

A magician has a stack of eight cards: 1 king of hearts, 1 queen of spades, 2 ten of spades, and 4 two of hearts. The magician does a magic trick at a birthday party in which a child draws a two of hearts and then places it back in the stack. He selects another child to draw a second card.

Are the children’s selections dependent or independent? Why?

A mother washed and folded four sets of sheets for her young son and placed each set in a clothing bin. The little boy is about to make his bed and would like to use his racecar sheet set. He reaches into the bin, blindly selects a sheet set, and pulls out a light blue set. He puts the blue set back in the bin, and then reaches into the bin again to try to select the racecar sheet set.

Are his selections dependent or independent? Why?